CS105L: Discrete Structures
I semester, 2005-06

Homework # 2

Due before class on **Friday, August 19, 2005**

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August 19, 2005

Prove the recursion theorem:

Theorem. (***The Recursion Theorem.***) *If* a *is an element of a set* X *and if* f *is a function from* X *into* X *then there exists a function* u *from the set of natural numbers* $\mathbb{N}$ *into* X *such that* $u(0) = a$ *and such that* $u(n+1) = f(u(n))$ *for all* $n \in \mathbb{N}$.

Hint. Start by defining relations that have the property u is supposed to have. Determine a relation of minimum size from these and use the fact that it is minimum in size to prove that it is actually a function.

Addendum. Someone suggested that there is an "obvious" solution as follows: Set $u(0)$ to a, $u(1)$ to $f(a)$ and $u(n)$ to $f^n(a)$ i.e. f applied n times to a. This solution is correct but it uses the Axiom of Choice. Show that this soultion uses the Axiom of Choice and prove the Theorem without it by using the hint given above.