

TUTORIAL SHEET 9

1. How many n -digit decimal sequences (using the digits $0 \dots 9$) are there in which the digits 1, 2 and 3 all appear?
2. How many arrangements of 52 letters, 2 A 's, 2 B 's, 2 C 's, etc. are there with no pair of consecutive letters the same?
3. How many ways are there of dealing a 13 card hand with at least one void in a suit?
4. The Mobius function $\mu(n)$ is 0 if n contains a square factor and is $(-1)^r$ if n is the product of r different primes. For any $n \geq 2$, prove that $\sum_{d|n} \mu(d) = 0$.
5. If 8 dies are rolled what is the probability that all 6 numbers appear?
6. How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s ?
7. Use the product rule to show that there are 2^{2^n} different truth tables for propositions in n variables.
8. Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.
9. How many ways are there to distribute six indistinguishable objects into four indistinguishable boxes such that each of the boxes contains at least one object ?
10. Suppose that S is a set with n elements. How many ordered pairs (A, B) are there such that A and B are subsets of S with $A \subseteq B$? [Hint: Show that each element of S belongs to $A, B - A$, or $S - B$].