COL 202

TUTORIAL SHEET 6

1. Show that if a and b are positive integers, then $(2^a - 1)\texttt{mod}(2^b - 1) = 2^{a \texttt{ mod } b} - 1$.

2. Use the above to show that if a and b are positive integers, then $\texttt{gcd}(2^a - 1, 2^b - 1) = 2^{\texttt{gcd}(a,b)} - 1$. [Hint: Show that the remainders obtained when the Euclidean algorithm is used to compute $\texttt{gcd}(2^a - 1, 2^b - 1)$ are of the form $2^r - 1$, where r is a remainder arising when the Euclidean algorithm is used to find $\texttt{gcd}(a, b)$.]

3. Prove or disprove: $p_1 p_2 \dots p_n + 1$ is prime for every positive integer n, where $p_1, \dots, p_n$ are the n smallest prime numbers.

4. Prove the correctness of the following rule to check if a number N is divisible by 7: Partition N into 3 digit numbers from the right $(d_3d_2d_1, d_6d_5d_4, \dots)$. The alternating sum $(d_3d_2d_1 - d_6d_5d_4 + d_9d_8d_7 - \dots)$ is divisible by 7 if and only if N is divisible by 7.

5. How many zeros are there at the end of 100! ?

6. Show that $\log_2 3$ is an irrational number.

7. Prove or disprove: there are three consecutive odd primes, i.e., odd primes of the form $p, p + 1, p + 4$.

8. Prove that the set of positive rationals is countable by showing that the following function K is a 1-1 correspondence between the set of positive rational numbers and the set of positive numbers: $K(m/n) = p_1^{2a_1} p_2^{2a_2} \dots p_s^{2a_s} q_1^{2b_1-1} q_2^{2b_2-1} \dots q_t^{2b_t-1}$, where $\texttt{gcd}(m, n) = 1$ and the prime power factorization of m and n are $p_1^{a_1} p_2^{a_2} \dots p_s^{a_s}$ and $q_1^{b_1} q_2^{b_2} \dots q_t^{b_t}$ respectively.