COL 202

TUTORIAL SHEET 5

1. Determine whether each of the following sets is countable or uncountable:
   - all positive rational numbers that cannot be written with denominators less than 4.
   - real numbers not containing 0 in their decimal representation
   - real numbers containing only finite number of 1's in their decimal representation
2. Show that if A is an infinite set, then it contains a countably infinite set.
3. Show that the union of a countable number of countable sets is countable.
4. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes out socks (in dark) in some arbitrary order.
   - How many socks he must take out to be sure that he has two socks of the same color ?
   - How many socks he must take out to be sure that he has two black socks ?
5. Let $(x_i, y_i), i = 1, \ldots, 5$ be a set of 5 distinct points with integer coordinates in the plane. Show that the mid-point of the line joining at least one pair of points has integer coordinates.
6. Suppose we color the plane with two colors – blue and red (so every point in the plane is colored either blue or red). Show that there exist two points in the plane which are exactly 1 meter apart and have the same color.
7. Let x be an irrational number. Given a positive integer n, show that there is a positive integer j not exceeding n, such that the absolute value of the difference between jx and the nearest integer to jx is less than $1/n$.
8. Prove that given any set of distinct 52 positive integers, there exist two integers m, n in this set such that either $m + n$ or $m - n$ is divisible by 100.