COL 202

TUTORIAL SHEET 11

1. Solve the following recurrence relations:
   - $a_r^2 - 2a_{r-1}^2 = 1, a_0 = 2.$
   - $ra_r + ra_{r-1} - a_{r-1} = 2^r, a_0 = 273.$
   - $a_r^2 - 2a_{r-1} = 0, a_0 = 4.$
   - $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + \cdots}}, a_0 = 4.$
   - $a_r - ra_{r-1} = r!, r = 1.$

2. Solve the recurrence relation $T(n) = nT^2(n/2)$ with initial condition $T(1) = 6$ when $n = 2k$ for some integer k. Hint: Let $n = 2^k$ and then make the substitution $a_k = \log T(2^k)$ to obtain a linear non-homogenous recurrence relation.

3. Show that the recurrence relation

$$f(n)a_n = g(n)a_{n-1} + h(n)$$

   for $n \geq 1$ with $a_0 = C$ can be reduced to a recurrence relation of the form

$$b_n = b_{n-1} + Q(n)h(n)$$

   where $b_n = g(n+1)Q(n+1)a_n$, with $Q(n) = (f(1)f(2)\ldots f(n-1))/(g(1)g(2)\ldots g(n))$. Use this to solve the original recurrence relation to obtain $a_n = \frac{C+\sum_{i=1}^{n} Q(i)h(i)}{g(n+1)Q(n+1)}$.

4. Use Q3 to solve the recurrence relation $(n+1)a_n = (n+3)a_{n-1} + n$, for $n \geq 1$ with $a_0 = 1$.

5. Show that if $a_n = a_{n-1} + a_{n-2}, a_0 = s$ and $a_1 = t$, where s and t are constants, then $a_n = sf_{n-1} + tf_n$ for all positive integers n where f_n is the n^{th} fibonacci number.

6. Solve the simultaneous recurrence relations

$$a_n = 3a_{n-1} + 2b_{n-1}, b_n = a_{n-1} + 2b_{n-1}$$

   with $a_0 = 1, b_0 = 2.$