COL 202

TUTORIAL SHEET 10

1. Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive 0s or two consecutive 1s.

2. Find a recurrence relation for the number of bit strings of length n that contain the string 01.

3. Find the recurrence relation satisfied by R_n, where R_n is the number of regions that a plane is divided into by n lines, if no two of the lines are parallel and no three of the lines go through the same point.

4. Let A_n be the $n \times n$ matrix with 2's on its main diagonal, 1's in all positions next to a diagonal element, and 0's everywhere else. Find a recurrence relation for d_n, the determinant of A_n. Solve this recurrence relation to find a formula for d_n.

5. Let $S(m, n)$ denote the number of onto functions from a set with m elements to a set with n elements. Show that $S(m, n)$ satisfies the recurrence relation $S(m, n) = n^m - \sum_{k=1}^{n-1} C(n, k)S(m, k)$ whenever $m \geq n$ and $n \geq 1$, with the initial condition $S(m, 1) = 1$.

6. Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and n as their last term, where n is a positive integer.

7. Find a recurrence relation for the number of ternary strings that do not contain two consecutive 0s or two consecutive 1s.

8. Find a recurrence relation for the number of ternary strings that contain two consecutive symbols that are the same.